Evaluation of Students' Mathematics Understanding in Mathematical Statistics

Adel Ali Diab [1], Fatima Emhemed Salem [2]

[1,2] *Mathematics Dept-Faculty of Science,* Bani Waleed *University, Libya*

[1] adeldiab812@gmail.com

[2] fatima3121982@gmail.com

Submitted: 17- Mar - 2024

Accepted: 25- Mar - 2024

Published: 27-Mar - 2024

ABSTRACT:

Students will achieve a higher cognitive level if they have a thorough knowledge of the scientific idea being studied. As a result, it is vital to conduct studies to better comprehend the concept of pupils in the science under consideration. The quantitative descriptive approach will be used to research third-level students in Mathematical Statistics, Mathematics Education Study Program, University Academic Year 2022-2023. A test instrument is used to perform data collection activities. Results of this study: 1) In general, students exhibit strong mathematical comprehension in Mathematical Statistics. Course 2) Most students still struggle to solve issues relating to the necessary subject, in this example involving infinite geometric series, the usage of logarithmic characteristics, integration procedures, or differentiation. 3) Algebraic operations are the most typical mistake that students make when settling. It is envisaged that this research will result in improvements to curriculum, instructional techniques, and teaching materials.

Keywords: Analysis, Understanding Mathematics, Mathematical Statistics.

Introduction

Students should be able to comprehend the material being taught, not only memorize it and then forget it, according to one of the learning objectives. Students will reach a higher cognitive level if they have a solid conceptual knowledge of the science they have studied. Pupils are able to make associations between different scientific concepts being taught and notions from other domains of science. Aside from that, pupils who grasp the material well are able to work through difficulties. Consequently, in order to gauge how well students have internalized the information presented in lectures, it is vital to examine their mathematical comprehension of the scientific subject they are learning. to ascertain the degree of comprehension, mistakes, and challenges that students face throughout the learning process.

The process of comprehending mathematical concepts might be defined as understanding mathematics. According to Bloom, understanding falls into the second category of cognitive abilities. This category comprises the following abilities: 1) Conceptual understanding; 2) Principle, rule, and generalization understanding; 3) Understanding of mathematical structures; 4) Transformational ability; 5) Ability to follow thought patterns; and 6) Ability to read and interpret mathematical or social problems.

Then, Wijaya presented three categories of mathematical comprehension abilities, which are as follows: 1) Translation, or conversion, is the process of assigning meaning to a variety of data in order to transmit information in other languages and formats. 2) Interpretation: interpretation is the process of giving reading meaning. It encompasses more than just understanding information derived from words and phrases; it also involves understanding information included in an idea. 3) Extrapolation: according to Wijaya, extrapolation comprises conclusions with implications based on knowledge at the third cognitive level, or application. It also involves estimations and forecasts that are based on an idea and a description of the information's conditions.

Research methods

The research method used in this research is a quantitative descriptive research method. This aims to find out and examine mathematical understanding abilities, difficulties, and errors in solving questions in the Mathematical Statistics I course, especially material on probability, probability density functions, and expectations on discrete distributions and continuous distributions. The research subjects were level III students in the Mathematical Statistics I Mathematics Education Study Program, Bani Waleed University, Academic Year 2022-2023. The research instrument used was the test instrument used, which was the test questions in the Mathematical Statistics I course regarding probability material, probability density functions, and expectations in discrete and continuous distributions. Data analysis using quantitative descriptive analysis in analyzing written test results data on students' mathematical understanding abilities.

Results and discussion

The outcomes of the paper assessments in the Mathematical Statistics I course, which covers content on probability, probability density functions, and expectations on discrete and continuous distributions, provide information on students' mathematical understanding ability scores. Six descriptive questions make up the exam, which assesses pupils' mathematical proficiency.

Students Mathematical Understanding Ability

The three categories of mathematical understanding abilities—translation, interpretation, and extrapolation—proposed by Wijaya are the indications used to gauge pupils' proficiency in mathematics. The following are the outcomes of data processing descriptive statistics test scores from descriptive instruments for students' mathematics comprehension skills.

Table 1. Descriptive Statistics of Mathematical Understanding Ability Scores

	N	Minimum	Maksimum	Mean	Variansi
Skor	23	19	29	22.91	8.083

With an average score of 22.91 and a variance of 8,083, Table 1 reveals that out of the 23 study subjects with an ideal score of 30, the lowest score derived from the results of the description test was 19 and the highest score was 29.

Meanwhile, for processing the description test data to measure mathematical understanding using Microsoft Excel 2013. From Table 2, it can be seen that the student's overall mathematical understanding ability reached 76.377%. So it can be concluded that more than half of all students can understand the material provided. However, it can also be seen that there are still weaknesses in the translation indicator with a value of 58.261%.

The following are the results of data processing on students' mathematical understanding abilities.

Table 2. Percentage of Students' Mathematical Understanding Ability

No	Indicators of Mathematical Understanding Ability	Percentage
1	Translation	58,261 %
2	Interpolation	76,087 %
3	Extrapolation	82,478 %
Total		76,377 %

The percentage of students who achieved the required level of mathematical understanding for each question on the test is then shown below, with data processing done using Microsoft Excel 2016.

Table 3. Percentage of Achievement of Mathematical Understanding Ability

No	Question	Achievement Level
1	Determine the value of the constant c from the pmf function of a discrete random variable X $$p_x(x) = c\left(\frac{2}{3}\right)^x \quad \text{dengan } x = 1, 2, 3, 4, \ldots$$	62,319 %
2	In a radioactive experiment, the average amount of radioactivity counted is 2 radioactive perms. Determine the probability of counting 1 radioactive perms.	90,271 %
3	Suppose that X is a discrete random variable with a binomial distribution with parameters n and p = 0.5. Determine the smallest integer n such that	59,420 %

4	Given a random variable X, the number of patients who recover from a disease D, with a 60% chance of recovery. Determine the probability that out of 5 patients with disease D, fewer than 4 patients will recover.	83,478 %
5	Determine from a pdf random variable X $$f_X(x) = \frac{x+2}{18} \quad \text{dengan } -2 < x < 4.$$	79,130 %
6	For example, the heights of 1000 25-year-old people surveyed are normally distributed with a mean of 160 cm and a variance of 25 cm. From that person, decide Many people are less than 160 cm tall. Many people are 150 to 170 cm tall. Ket:$\Phi(0) = 0.5, \Phi(1) = 0.841, \Phi(2) = 0.977 \quad \Phi(3) = 0.998$	96,739 %
Total		76,377 %

In Table 3 it can be seen that the majority of students can understand the material provided well. However, it can be seen that for questions number 1 and number 3, students have difficulty giving answers correctly, this could be due to a lack of understanding of the prerequisite material, especially Calculus.

The following are some of the results of student work related to mathematical understanding abilities.

In Answer 1, students experience errors in the translation indicators. In many answers, students are unable to convert geometric series into equivalent forms, so the final answer obtained is not correct even though students have completed the e algebraic operations well.

<u>Answer 1</u>

Student's Wrong Answer to Question Number 1

$$1. P_x(x) = c\left(\frac{2}{3}\right)^x \quad x = 1, 2, 3, 4, \dots \dots \dots$$

$$\sum_{i=1}^{\infty} c\left(\frac{2}{3}\right)^x = 1$$

$$c\left(\frac{2}{3}\right)^1 + c\left(\frac{2}{3}\right)^2 + c\left(\frac{2}{3}\right)^3 + c\left(\frac{2}{3}\right)^4 + \cdots + \cdots = 1$$

$$c(a^1 + a^2 + a^3 + \cdots) = 1$$

$$c\left(\left(\frac{2}{3}\right)^1 + \left(\frac{2}{3}\right)^2 + \left(\frac{2}{3}\right)^3 + \left(\frac{2}{3}\right)^4 \dots\right) = 1$$

$$c\left(\frac{1}{1-\frac{2}{3}}\right) = 1$$

$$c\left(\frac{1}{\frac{1}{3}}\right) = 1$$
$$3\ c = 1$$
$$c = \frac{1}{3}$$

In the meantime, students made numerous mistakes in the translation and interpolation signs in number 3. An illustration of a student's response to question number three is shown below.

Student's Wrong Answer to Question Number 3.

$$\begin{aligned}
&p(x \geq 1) \geq 0.85 \\
&P_x(u) = \binom{n}{u}\ p^u(1-p)^{n-u} \\
&\quad = \binom{n}{u}\ (0.5)^x(1-0.5)^{n-u} \\
&\quad = \binom{n}{1}\ (0.5)^1(1-0.5)^{n-1} \\
&\quad = \frac{n!}{1!\ (n-1)!}\ (0.5)^1\ (0.5)^{n-1} \\
&\quad = \frac{n!}{(n-1)!}\ (0.5)^n = 0.85 \\
&\quad = \frac{n!\,(n-1)!}{(n-1)!} = 0.85 \\
&\quad = \ n(0.5) = 0.85 \\
&misol\ \ (0.5)^n = \frac{0.85}{n} \\
&\quad a^x = \mathrm{b}\ \ misol\,\mathrm{a} = 0.5\ \ ;\mathrm{x} = \mathrm{n}\,;\mathrm{b} = \frac{0.85}{n} \\
&\quad a\log \quad b = c \\
&\quad 0.5\log\ \ \frac{0.85}{n} = c \\
&\quad \mathrm{c} = 0
\end{aligned}$$

Students are not yet able to identify or relate the probability of a binomial distribution whose value is known to the number of experiments that must be carried out to obtain the probability value. Then many students cannot change logarithmic values into other values that are equivalent to them. So the student's answer is incorrect or the student can no longer continue his answer.

Difficulties Experienced by Students in Solving Questions in Mathematical Statistics I

The difficulties experienced by students in solving questions vary. Therefore, each number of difficulties experienced by students will be analyzed in more detail, with the aim being that students will be able to answer questions on the same material better.

In terms of the Mathematics Statistics I material presented, students still have many difficulties if they get questions that are not routinely done or are related to prerequisite subjects, especially Calculus. Based on the results in Table 3, the description questions can be grouped into easy questions, medium questions, and difficult questions. Easy questions on numbers 2 and 6, medium questions on numbers 4 and 5, and difficult questions on questions 1 and 3.

In question number 1, students are expected to be able to solve the problem of the probability density function of a given discrete random variable. In number 1, most students have difficulty solving the infinite geometric series needed to find the value of the constant c in question. When an infinite geometric series cannot be changed into another equivalent form, then students cannot work on it or even think that this problem cannot be solved. The following is an example of a student's answer to number 1 which will be presented below.

Example of a student's wrong answer for number 1.

$$1. P_x(x) = c\left(\frac{2}{3}\right)^x \quad x = 1, 2, 3, 4, \dots\dots\dots$$

$$\sum_{i=1}^{\infty} c\left(\frac{2}{3}\right)^x = 1$$

$$c\left(\frac{2}{3}\right)^1 + c\left(\frac{2}{3}\right)^2 + c\left(\frac{2}{3}\right)^3 + c\left(\frac{2}{3}\right)^4 + \cdots + \cdots = 1$$

$$c(a^1 + a^2 + a^3 + \cdots) = 1$$

$$c\left(\left(\frac{2}{3}\right)^1 + \left(\frac{2}{3}\right)^2 + \left(\frac{2}{3}\right)^3 + \left(\frac{2}{3}\right)^4 \dots\right) = 1$$

$$c\left(\frac{1}{1-\frac{2}{3}}\right) = 1$$

$$c\left(\frac{1}{\frac{1}{3}}\right) = 1$$

$$3\ c = 1$$

$$c = \frac{1}{3}$$

In question number 2, most students did not experience significant difficulties. Students simply enter the x value they want to search for into their probability density function. However, some students have difficulty identifying the correct probability density function, so they use the incorrect probability density function, which, in the end, will produce an incorrect answer even though the algebraic operations are correct. The following is an example of a student's answer to number 2, which will be presented below.

Example of a student's correct answer for number 2.

$$Sis = x \sim \; Poisson\, Distribution$$
$$rata - rata \quad = \lambda \; = 2$$
$$Sis\!: a) \;\; p(x = 1) = \cdots \quad , b) \;\; p(x \leq 2) = \cdots$$
$$Answer\!: \;\; a) \;\; p(x = 1) = \frac{\lambda^{\varphi} e^{-x}}{\varphi!} = \frac{2^1 e^{-2}}{1!} = \frac{2. e^{-2}}{1!} = 2. e^{-2} = 0.27$$

On question number 3, almost half of the students experienced difficulty. Almost all the difficulties experienced are related to students' inability to use the concept of logarithms, namely to change the logarithmic equation in the problem into another form that is equivalent and simpler. Logarithm material is prerequisite course material. The following is an example of a student's answer to number 3 which will be presented below.

Example of a student's wrong answer for number 3.

$$p(x \geq 1) \geq 0.85$$
$$P_x(u) = \binom{n}{u} \; p^u (1-p)^{n-u}$$
$$= \binom{n}{u} \; (0.5)^x (1-0.5)^{n-u}$$
$$= \binom{n}{1} \; (0.5)^1 (1-0.5)^{n-1}$$
$$= \frac{n!}{1! \; (n-1)!} \;\; (0.5)^1 \; (0.5)^{n-1}$$
$$= \frac{n!}{(n-1)!} \;\; (0.5)^n = 0.85$$
$$= \frac{n! \, (n-1)!}{(n-1)!} = 0.85$$
$$= \;\; n(0.5) = 0.85$$
$$misol \;\; (0.5)^n = \frac{0.85}{n}$$
$$a^x = \mathrm{b} \quad misol \; \mathrm{a} = 0.5 \;\; ; \mathrm{x} = \mathrm{n} \; ; \mathrm{b} = \frac{0.85}{n}$$
$$a \log \quad b = c$$
$$0.5 \log \;\; \frac{0.85}{n} = c$$
$$\mathrm{c} = 0$$

In question number 4, students generally did not experience any difficulties. However, some students experience difficulty defining the probability density function that will be used to solve the problem. The probability density function that should be used is binomial; many students use other probability density functions. So that in the next process, it will produce an incorrect answer, even though, algebraically speaking, their answer is correct. The following is an example of a student's answer to number 4, which will be presented below.

Example of a student's correct answer for number 4.

$Sis\ :\ \ x \sim the\ number\ of\ patients\ who\ recover\ after\ being\ attacked\ by\ the\ disease\ \ D$.

$x \sim\ Binomial$

$p = 60\% \ = 0.6 \quad . \quad n = 5$

$Sis\ :\ a)\, p(x < 4) = \cdots \quad b) \quad p(x \geq 4) = \cdots$

$Answer\!: a)\ \ p(x < 4) = \ p(x = 1) + p(x = 2) + p(x = 3)$

$$= \binom{5}{1}\,(0.6)^1\,.\,(0.4)^4 + \binom{5}{2}\,(0.6)^2\,.\,(0.4)^3 + \binom{5}{3}\,(0.6)^2\,.\,(0.4)^2$$

$= 5.\,(0.6).\,(0.0256) + 10.\,(0.36).\,(0.064) + 10(0.216).\,0.16$

$= 5.\,(0.01536) + 10(0.02304) + 10(0.03456)$

$= 0.0765 + 0.2304 \ \ , \qquad 0.3456$

$= 0.6528$

In question number 5, students generally understand the expected material, but the difficulty that occurs is when students have to complete the integral of the given function to get the expected value. Integral engineering is prerequisite course material. The following is an example of a student's answer to number 5, which will be presented below.

Example of a student's wrong answer for number 5.

5. $\quad f_x(x) = \dfrac{x+2}{18} \qquad -2 < x < 4$

$$a.\ \ E[x] \ = \int_{-2}^{4} x\,.\,f_x\ dx$$

$$= \int_{-2}^{4} x\,.\frac{x+2}{18}\ dx$$

$$= \int_{-2}^{4} \frac{x^2+2x}{18}\ dx$$

$$= \frac{x^3}{54} + \frac{2x^2}{36} \quad \Big]_{-2}^{4}$$

$$= \left(\frac{64}{54} + \frac{32}{36}\right) - \left(\frac{-8}{54} + \frac{8}{36}\right) = \frac{72}{54} + \frac{24}{36} = \frac{2592 + 1296}{1944} = 2$$

Finally, on question number 6, almost all students were able to do this question well, but for some students, the difficulty that occurred was in understanding the question given. Students did not understand the problem completely, so the answer given did not fully solve the problem. It takes a few more steps to solve the problem. The following is an example of a student's answer to number 6, which will be presented below.

Overall, it can be concluded from the six questions given that students have mastered the concepts of probability, probability density function, and expectations in discrete and continuous distributions. The difficulties experienced by students are mostly related to prerequisite material such as how to calculate infinite geometric series and integration techniques, as well as some difficulties in precisely defining the probability density function that will be used in solving problems.

Example of a student's almost correct answer for number 6.

6. $\mu = 160 \quad \sigma^2 = 25 \quad \rightarrow \quad \vartheta = 5$

$x.Normal \; (160\,,5)$

$$
\begin{aligned}
\text{a. } \mathrm{p}(\mathrm{x} < 160) &= \mathrm{p}(x - \mu < 160 - \mu) \\
&= \mathrm{p}\left(\frac{x-\mu}{\vartheta} < \frac{160-\mu}{\vartheta}\right) \\
&= \mathrm{p}\left(z < \frac{160-160}{5}\right) \\
&= \mathrm{p}(z < 0) \\
&= \varphi(0) \\
&= 0.5
\end{aligned}
$$

$$
\begin{aligned}
\text{b. } \mathrm{p}(150 < \mathrm{x} < 170) &= \mathrm{p}(x < 150) - \mathrm{p}(x < 170) \\
&= \varphi\left(\frac{150-\mu}{\vartheta}\right) - \varphi\left(\frac{170-\mu}{\vartheta}\right) \\
&= \varphi\left(\frac{150-160}{5}\right) - \varphi\left(\frac{170-160}{5}\right) \\
&= \varphi\left(\frac{-10}{5}\right) - \varphi\left(\frac{10}{5}\right) \\
&= \varphi(-2) - \varphi(2) \\
&= \big(1 - \varphi(2)\big) - \varphi(2) \\
&= (1 - 0.9772) - 0.9772 \\
&= 0.0228 - 0.9772 = -0.9544
\end{aligned}
$$

The following is the level of difficulty of the questions worked on by students based on an analysis of the level of students' mathematical understanding abilities which is presented in Table3.

Errors Experienced by Students in Solving Questions in Mathematical Statistics I

Students' mistakes in solving questions revolve around the same problem and occur repeatedly. Most mistakes are due to not understanding the concepts of the prerequisite material. Then, other mistakes that often occur and are repeated are when carrying out algebraic operations (addition, subtraction, multiplication, division), exponentiation and root operations, and logarithms. Apart from that, several errors occurred in writing the mathematical notation.

In the process of identifying student errors in doing the work given, errors will be divided into several types, including errors in algebraic operations as well as errors in integration techniques, use of logarithm properties, determining series, or those related to prerequisite material, such as notation writing errors. mathematics, errors in understanding Mathematical Statistics I material, and other errors. In Table 4, the mistakes that students often experience are presented.

Table 4 Errors in answering questions

No	Error Type	Number	Percentage
1	Prerequisite Material	18	56.350 %
2	Notation Writing	5	15.625 %
3	StatMat I material	7	21.875 %
4	Other	2	6.250 %

Table 4 shows that the types of errors experienced by students mostly occurred in errors in working on prerequisite material, namely 56,350%, followed by errors in understanding concepts of Mathematics Statistics I material, and errors in writing notation, which amounted to 21,875% and 15,625% respectively. Below are examples of answers from students who made errors in calculating algebraic operations related to prerequisite material.

Example of a student's answer who made an algebra operation error

$$p(x \geq 1) \geq 0.85$$

$$\begin{aligned}
P_x(u) &= \binom{n}{u} p^u (1-p)^{n-u} \\
&= \binom{n}{u} (0.5)^x (1-0.5)^{n-u} \\
&= \binom{n}{1} (0.5)^1 (1-0.5)^{n-1} \\
&= \frac{n!}{1!\,(n-1)!} (0.5)^1 (0.5)^{n-1} \\
&= \frac{n!}{(n-1)!} (0.5)^n = 0.85 \\
&= \frac{n!\,(n-1)!}{(n-1)!} = 0.85 \\
&= n(0.5) = 0.85
\end{aligned}$$

$$misol\ (0.5)^n = \frac{0.85}{n}$$

$$a^x = \mathrm{b} \quad misol \ \mathrm{a} = 0.5 \ ; \mathrm{x} = \mathrm{n} \ ; \mathrm{b} = \frac{0.85}{n}$$

$$a \log \quad b = c$$

$$0.5 \log \quad \frac{0.85}{n} = c$$

$$\mathrm{c} = 0$$

Examples of answers from students who make mistakes in algebra operations.

$$5.a. \ E[x] \ = \int_{-2}^{4} x \, . F_x \ dx$$

$$= \int_{-2}^{4} x \, . \frac{x+2}{18} \ dx$$

$$= \int_{-2}^{4} \frac{x^2 + 2x}{18} \ dx$$

$$= \frac{\frac{x^3}{3} + x^2}{18x} \quad \Big]_{-2}^{4}$$

$$= \frac{\frac{(4)^3}{3} + (4)^2}{18(4)} - \frac{\frac{(-2)^3}{3} + (-2)^2}{18(-2)}$$

$$= \frac{\frac{64}{3} + 16}{72} - \frac{\frac{-8}{3} + 4}{-36}$$

$$= \frac{21.3 + 16}{72} - \frac{(-2.7) + 4}{-36}$$

$$= \frac{37.3}{72} - \frac{1.3}{-36} = 0.51 - (-0.03) = 0.54$$

Examples of Answers from Students Who Make Algebra Operation Errors

$$1. P_x(x) = c\left(\frac{2}{3}\right)^x \quad x = 1, 2, 3, 4, \dots \dots \dots$$

$$\sum_{i=1}^{\infty} c\left(\frac{2}{3}\right)^x = 1$$

$$c\left(\frac{2}{3}\right)^1 + c\left(\frac{2}{3}\right)^2 + c\left(\frac{2}{3}\right)^3 + c\left(\frac{2}{3}\right)^4 + \cdots + \cdots = 1$$

$$c(a^1 + a^2 + a^3 + \cdots) = 1$$

$$c\left(\left(\frac{2}{3}\right)^1 + \left(\frac{2}{3}\right)^2 + \left(\frac{2}{3}\right)^3 + \left(\frac{2}{3}\right)^4 \dots\right) = 1$$

$$c\left(\frac{1}{1 - \frac{2}{3}}\right) = 1$$

$$c\left(\frac{1}{\frac{1}{3}}\right) = 1$$
$$3\ c = 1$$
$$c = \frac{1}{3}$$

Conclusion

Based on the results and discussion in the previous section, it can be concluded that:

1. Most students already have good mathematical understanding skills in Mathematical Statistics I.
2. Most students still have difficulty solving questions related to prerequisite material, in this case about infinite geometric series, the use of logarithmic properties, and integration techniques.
3. The errors that most often occur in solutions made by students are algebraic operations.

Recommendation

Then, based on the conclusions above, the researcher feels it is necessary to provide suggestions according to the results and discussion of the research. The suggestions given include:

1. Prerequisite knowledge related to Mathematical Statistics should be presented in full.
2. Providing repeated practice questions can familiarize students with overcoming difficulties and mistakes they experience.
3. Try several alternative learning models so that students can better understand the Mathematical Statistics material.

Rreferences

Wijaya, R. P., Budiarto, M. T., Wijayanti, P. (2021). Development of realistic mathematics learning tools to improve students' mathematical literacy ability. Mathematics Education Journals, 5(2), 124-131.

Yeo, J. B. (2007). Mathematical tasks: Clarification, classification and choice of suitable tasks for different types of learning and assessment. Mathematics and Mathematics Education National Institute of Education Technical Report ME2007-01

Zulkardi, Putri, R. I. I., Alwi, Z., Nusantara, D. S., Ambarita, S. M., Maharani, Y., & Puspitasari, L. (2020). How students work with PISA-like mathematical tasks using COVID-19 context. Journal on Mathematics Education, 11(3), 405-416.

Rakhmawati, Y., & Mustadi, A. (2022). The circumstances of literacy numeracy skill: Between notion and fact from elementary school students. Jurnal Prima Edukasia, 10(1), 9- 18.

Plomp, T., & Nieveen, N. (2007). An Introduction to Educational Design Research. In Proceedings of the Seminar Conducted at the East China Normal University [Z]. Shanghai: SLO-Netherlands Institute for Curriculum Development.

Pepin, B. (2015). Enhancing mathematics/STEM education: A ‘resourceful’ approach. Inaugural lecture, 27 November 2015, Technische Universiteit Eindhoven