Optimization of Coupled harmonic oscillators for Defining One leg Locomotion

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Abstract: With regard to the dynamics of a coupled harmonic oscillators ground and coherent state are explored. The approach is focused on solving for the time evolution operator and then applying it to a tensor product of a ground and coherent state representing a physical system and environment respectively. The coherent state is then partially traced to extract the dynamics of the ground state. The time evolution operator is found by solving a series of eleven coupled differential equations. Also, this study will discuss how to optimize the parameter of coupled harmonic oscillators to generate arithmetic motion for one leg with two degrees of freedom by using a genetic algorithm (GA). The results demonstrate that a change in coupling results in a change in the evolution of the ground state.

Keywords: Coupled harmonic oscillators, mathematical analysis, mathematical modelling, genetic algorithm.

1. Introduction

One of the most important topics is locomotion of robots, which is widely discussed recently. In biological systems, as many researches are shown that, a number of various patterns occur which are produced by a central nervous system such as, running, walking, swimming and crawling. The central nervous system is called Central Pattern Generator (CPG) and here this study, we will use couple harmonic oscillators instead of CPGs. According to these researches these CPGs or oscillators are exited or cited in the locals of the spinal cord as shown by some researches [1], [2], [3] and[4] .

Biologically, CPGs can be defined as inspired networks of nonlinear oscillating neurons capable of generating rhythmic patterns without higher control centres or sensory feedback [3] and [5]. A neural oscillator comprises a pair of neurons with inhibitive connections between them. Each neuron suppresses the other, which are a flexor and an extensor neuron [6].

Various mathematical and physical structures of the legs and limbs have been modelled [7] and the control systems have been copied to reproduce patterns of movement in robots similar to those in certain biological organisms.

Interestingly, various mathematical structures in the literature have modelled and mimicked biological control parameters [7]. Various CPG models, including the connectionist models, have been implemented in the robotic systems [8]. Also, there are some studies discussing how systems of oscillators can be coupled [9].

we refer the reader to [10]. Some of these studies have been considered the Van der Pol and the Hopf oscillator. Others used different mathematical structures of CPGs to control bipedal locomotion.

This paper mainly focuses on the analysis and optimization of the Couple harmonic oscillators in details for the purpose of locomotion. This paper draws on and derives support from the studies mentioned above and investigates how Couple harmonic oscillators can be optimized for one leg with two degree of freedoms DOFs via an adaptation to the robotic systems that perform one leg movement. In particular, this paper investigates a nonlinear limit-cycle oscillator similar to those are discussed by [3], [11]. The mathematical analysis for the optimization of the CPG can be another novelty in this paper. Based on the cost function, this paper uses a developmental algorithm to find the optimum parametric values for couple harmonic oscillators.

The paper is organized as follows: The kinematic model has been discussed in the next section. Couple harmonic oscillators are given in Section 3. In Section 4, the numerical solution is discussed. Section 5 is devoted to the optimization results. In Section 6, some conclusions are drawn and suggestions for future research are given.

2. Kinematic Modeling of One Leg

In order to determine the kinematic attributes with the system behaviour while walking. In the figure 1 illustrates how couple harmonic oscillators are used to produce rhythmic patterns for the hip and knee via one leg of a human when the lower body is parallel to the ground. It is worth mentioning here that the results obtained are contingent upon the manner in which CPGs are analyzed.

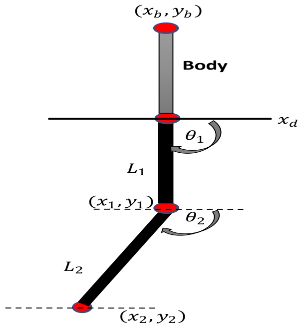

Figure 1: The Planar biped model when the lower body is parallel to the ground

A closer look into the kinematics of the hip and knee angles in the swing phase reveals the following basic kinematics equations: From the joint between hip and knee, we have. The first coordinate (x_1, y_1) yields

$$x_1 = x_d + L_1 cos\theta_1 \quad \text{and} \quad y_1 = y_d + L_1 sin\theta_1.$$

The second coordinate (x_2, y_2) reveals that

$$x_2 = x_d + L_1 cos\theta_1 + L_2 cos\theta_2$$

And

$$y_2 = y_d + L_1 sin\theta_1 + L_2 sin\theta_2,$$

where x_d is the proceeding displacement (i.e., the distance during locomotion) and y_d stands for the positive direction of the hip height at each step. L_1 and $L_{2,}$ represent three lengths: from the hip joint to the knee joint, from the knee joint to the end effector, respectively. The angles, θ_1 and θ_2 which represent the hip and knee, respectively, will acquire their rhythmic patterns from Couple harmonic oscillators. With regard to y_d , it is assumed to be zero when the lower body is parallel to the ground. The researchers in this study, however, fixed the hip joint in spite of the fact that the hip joint was not fixed when collecting real data.

3. Couple harmonic oscillators and it analysis

To discuss Equations of motion for forced vibration: -

By considering the a viscously damped with two DOF (degree of freedom) spring mass system as shown in Figure 2, the Motion is described by x_1 and x_2. Also, the m_1 and m_2 are represented the mass system at any time t.

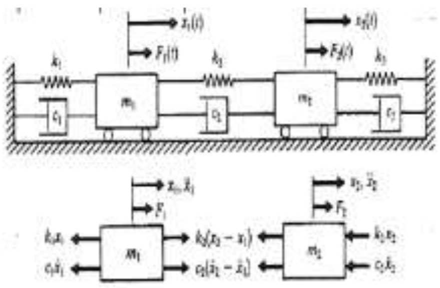

Figure 2: Spring-mass system

Let us assume that there are two forces F_1 and F_2 act on our model, by the Newton's second law of motion impact on the masses, we obtain

$$\left.\begin{aligned} m_1\ddot{x_1} + (c_1 + c_2)\dot{x_1} - c_2\dot{x_2} + (k_1 + k_2)x_1 - k_2x_2 = F_1 \\ m_2\ddot{x_2} - c_2\dot{x_1} + (c_2 + c_3)\dot{x_2} - k_2x_1 + (k_2 + k_3)x_2 = F_2 \end{aligned}\right\} \quad (1)$$

This system can be written as

$$[M]\ddot{X} + [C]\dot{X} + [K]X = F$$

Where, $[M] = \begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix}$, $[C] = \begin{bmatrix} c_1 + c_2 & -c_2 \\ -c_2 & c_2 + c_3 \end{bmatrix}$,

$$[K] = \begin{bmatrix} k_1 + k_2 & -k_2 \\ -k_2 & k_2 + k_3 \end{bmatrix}, [X] = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}, [F] = \begin{bmatrix} F_1 \\ F_2 \end{bmatrix}$$

Notice that

The matrices are symmetric matrices

$$[M]^T = [M], [C]^T = [C], [K]^T = [K]$$

Analysis the system:

When the system is an undamped at $c_1 = c_2 = c_3 = 0,$ by setting $F_1 = F_2 = 0$ our system is reduced

$$\left.\begin{array}{r} m_1\ddot{x_1} + (k_1 + k_2)x_1 - k_2x_2 = 0 \\ m_2\ddot{x_2} - k_2x_1 + (k_2 + k_3)x_2 = 0 \end{array}\right\} \qquad (2)$$

It is needed to find if both masses are oscillated harmonically with same frequency and phase angle with different amplitude. We may have harmonic solution

$$\left.\begin{array}{l} x_1(t) = Acos(\omega t + \emptyset) \\ x_2(t) = Bsin(\omega t + \emptyset) \end{array}\right\} \qquad (3)$$

Where A and B are constants which denote amplitude of $x_1(t)$ & $x_2(t)$ respectively, ω is frequency and $\emptyset$ is phase angle. It is easily to show that eq (3) is solution of eq (2): by differentiate the system (3) we obtain System (4):

$$\left.\begin{array}{r} \left(\left(-m_1\omega^2 + (k_1 + k_2)\right)x_1 - k_2x_2\right) cos(\omega t + \emptyset) = 0) \\ \left(-k_2x_1 + \left(-m_2\omega^2 + (k_2 + k_3)\right)x_2\right)cos(\omega t + \emptyset) = 0 \end{array}\right\} \qquad (4)$$

The system (4) can be written

$$\left.\begin{array}{r} \left(-m_1\omega^2 + (k_1 + k_2)\right)x_1 - k_2x_2 = 0 \\ -k_2x_1 + \left(-m_2\omega^2 + (k_2 + k_3)\right)x_2 = 0 \end{array}\right\} \qquad (5)$$

The system (5) is algebraic homogenous equations, we have two unknow variable x_1 & x_2. One of the solutions is trivial solution $x_1 = x_2 = 0$, For nontrivial solution of x_1 & x_2 the determinant of x_1 & x_2 coefficients must be zero

$$\begin{vmatrix} -m_1\omega^2 + (k_1 + k_2) & -k_2 \\ -k_2 & -m_2\omega^2 + (k_2 + k_3) \end{vmatrix} = 0$$

$$(m_1m_2)\omega^4 - \{(k_1 + k_2)m_2 + (k_2 + k_3)m_1\}\omega^2 + \{(k_1 + k_2)(k_2 + k_3) - k_2^2\} = 0 \qquad (6)$$

When we solve for the frequency, the equation (6) is called characteristic equation has the following solution

$$\omega_1^2, \omega_2^2 = \frac{1}{2}\left\{\frac{(k_1 + k_2)m_2 + (k_2 + k_3)m_1}{m_1m_2}\right\} \pm \frac{1}{2}\left[\left\{\frac{(k_1 + k_2)m_2 + (k_2 + k_3)m_1}{m_1m_2}\right\}^2 - 4\left\{\frac{(k_1 + k_2)(k_2 + k_3) - k_2^2}{m_1m_2}\right\}\right]^{\frac{1}{2}} \qquad (7)$$

This is nontrivial harmonic solution when $\omega = \omega_1 = \omega_2$, the Eq(5) is homogenous, the ratios $r_1 = x_2/x_1$ and $r_2 = x_1/x_2$ can be found by $\omega^2 = \omega_1^2 = \omega_2^2$, from Eq(5) and (7), we obtain

$$r_1 = \frac{x_2}{x_1} = \frac{-m_1\omega_1^2 + (k_1 + k_2)}{k_2} = \frac{k_2}{-m_2\omega_1^2 + (k_2 + k_3)}$$

$$r_2 = \frac{x_1}{x_2} = \frac{-m_1\omega_2^2 + (k_1 + k_2)}{k_2} = \frac{k_2}{-m_2\omega_2^2 + (k_2 + k_3)}$$

Why we use ratios !! here just to mention we have four roots for Eq(5), according to that: the four solutions are

$$\overrightarrow{x_1} = \begin{pmatrix} x_{11} \\ x_{12} \end{pmatrix} = \begin{pmatrix} Acos(\omega_1 t + \emptyset_1) \\ r_1Acos(\omega_1 t + \emptyset_1) \end{pmatrix}$$

$$\overrightarrow{x_2} = \begin{pmatrix} x_{21} \\ x_{22} \end{pmatrix} = \begin{pmatrix} Bcos(\omega_2 t + \emptyset_2) \\ r_2Bcos(\omega_2 t + \emptyset_2) \end{pmatrix}$$

We have four independent solutions, which can be written as:

$$x_1 = x_{11} + x_{12} = Acos(\omega_1 t + \emptyset_1) + r_1Acos(\omega_1 t + \emptyset_1)$$
$$x_2 = x_{21} + x_{22} = Bcos(\omega_2 t + \emptyset_2) + r_2Bcos(\omega_2 t + \emptyset_2)$$

To find A and B by using initial conditions. By assuming the positon and veolcity are equal to zero.
Our case:

$$\left.\begin{array}{r} m_1\ddot{x_1} + b_1\dot{x_1} + k_1'x_1 + k_3x_2 = 0 \\ m_2\ddot{x_2} + b_2\dot{x_2} + k_2'x_2 + k_3x_1 = 0 \end{array}\right\} \Rightarrow$$

$$\left.\begin{array}{r} m_1\ddot{x_1} = -b_1\dot{x_1} - k_1'x_1 - k_3x_2 \\ m_2\ddot{x_2} = -b_2\dot{x_2} - k_2'x_2 - k_3x_1 \end{array}\right\} \quad \Rightarrow$$

$$\left.\begin{aligned}\ddot{x_1} &= -\frac{b_1}{m_1}\dot{x_1} - \frac{k_1'}{m_1}x_1 - \frac{k_3}{m_1}x_2 \\ \ddot{x_2} &= -\frac{b_2}{m_2}\dot{x_2} - \frac{k_2'}{m_2}x_2 - \frac{k_3}{m_2}x_1\end{aligned}\right\}$$

If k_1' & k_2' are constants and Figure 3 is shown simulation block diagram.

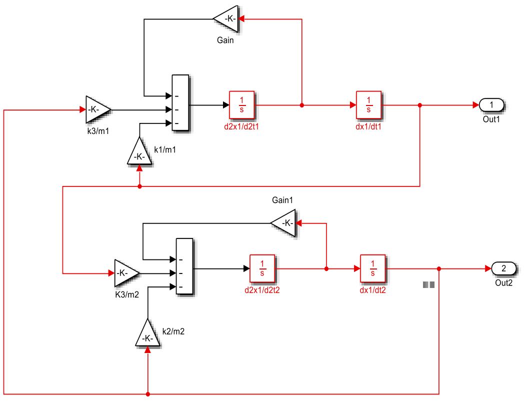

Figure 3: Simulation Model block diagram

Let us suggest and revise the above system as:

$$\left.\begin{aligned} m_1\ddot{x_1} + b_1\dot{x_1} + k_1'x_1 + k_3x_2 &= 0 \\ m_2\ddot{x_2} + b_2\dot{x_2} + k_2'x_2 + k_3x_1 &= 0\end{aligned}\right\} \Longrightarrow$$

$$\left.\begin{aligned} m_1\ddot{x_1} &= -b_1\dot{x_1} - k_1'x_1 - k_3x_2 \\ m_2\ddot{x_2} &= -b_2\dot{x_2} - k_2'x_2 - k_3x_1\end{aligned}\right\} \Longrightarrow$$

$$\left.\begin{aligned}\ddot{x_1} &= -\frac{b_1}{m_1}\dot{x_1} - \frac{k_1'}{m_1}x_1 - \frac{k_3}{m_1}x_2 \\ \ddot{x_2} &= -\frac{b_2}{m_2}\dot{x_2} - \frac{k_2'}{m_2}x_2 - \frac{k_3}{m_2}x_1\end{aligned}\right\}$$

where

$$(k_1 + k_2) = k_1'$$
$$(k_2 + k_3) = k_2'$$

To Find the numerical Solution, we need to reduce this system first order differential equations as:

$$y_1 = x_1 \Longrightarrow \dot{y_1} = y_2$$
$$y_2 = \dot{x_1} \Longrightarrow \dot{y_2} = -\frac{b_1}{m_1}y_2 - \frac{k_1'}{m_1}y_1 - \frac{k_3}{m_1}y_3$$
$$y_3 = x_2 \Longrightarrow \dot{y_3} = y_4$$
$$y_4 = \dot{x_2} \Longrightarrow \dot{y_4} = -\frac{b_2}{m_2}y_4 - \frac{k_2'}{m_2}y_3 - \frac{k_3}{m_2}y_1$$

New system is given as

$\dot{y_1} = y_2$

$\dot{y_2} = -\frac{b_1}{m_1}y_2 - \frac{k_1'}{m_1}y_1 - \frac{k_3}{m_1}y_3$

$\dot{y_3} = y_4$

$\dot{y_4} = -\frac{b_2}{m_2}y_4 - \frac{k_2'}{m_2}y_3 - \frac{k_3}{m_2}y_1$

$y_1 = 2,\quad y_2 = 0,\quad y_3 = 1.5,\quad y_4 = 0,$

New, let us consider different cases for analysis numerical solution.

First case: let us fixed $m_1 = m_2 = 10, k_1 = 140$; $k_2 = 140$; $k_3 = 140$: $b_1 = 2\sqrt{m_1k_1}$, and we also manipulate by changing $b_2 = 0.1 * b_1$. Numerical is as seen in the figure 4

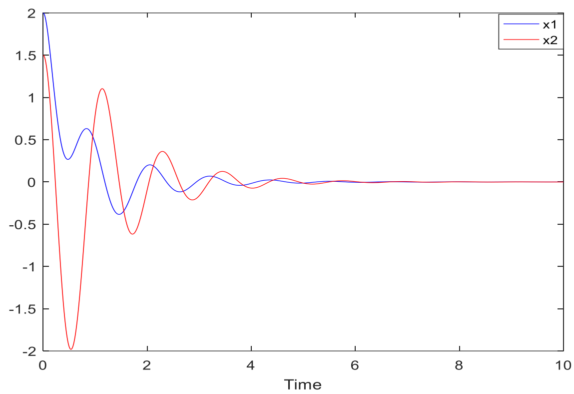

Figure 4: The Output of x_1 and x_2 corresponding to $m_1 = m_2 = 10, k_1 = 140$; $k_2 = 140$; $k_3 = 140, b_2 = 0.1 * b_1$

Note that, even if we play with both parameters $b_2 = 0.1 * b_1$ by increasing the relation's value or decrease, we will get under damped.

Second case: Let us assume again, $m_1 = m_2 = 4, k_1 = 120$; $k_2 = 120$; $k_3 = 120$; $b_1 = 2\sqrt{m_1 k_1}$, and we also manipulate by changing $b_2 = 0.1 * b_1$. Numerical is as seen in the figure 5.

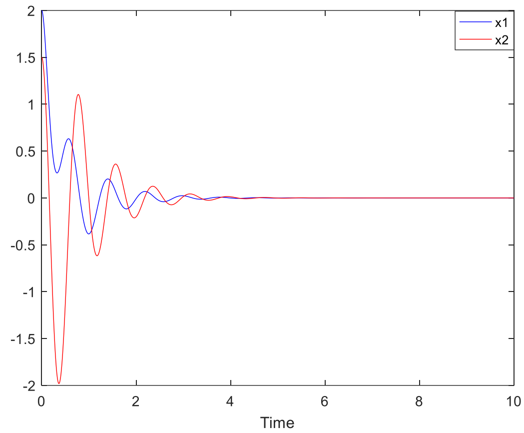

Figure: 5 The Output of x_1 and x_2 corresponding to $m_1 = m_2 = 4, k_1 = 120$; $k_2 = 120$; $k_3 = 120$; $b_1 = 2\sqrt{m_1 k_1}, b_2 = 0.1 * b_1$

Although, we start changing to parameters $b_2 = 0.1 * b_1$ by increasing decreasing we obtain also under damped.

Third case: Let us consider, $m_1 = m_2 = 2, k_1 = 24$; $k_2 = 24$; $k_3 = 24$; $b_1 = 2\sqrt{m_1 k_1}$, we also manipulate by changing $b_2 = 0.001 * b_1$. Numerical is as seen in the figure 6.

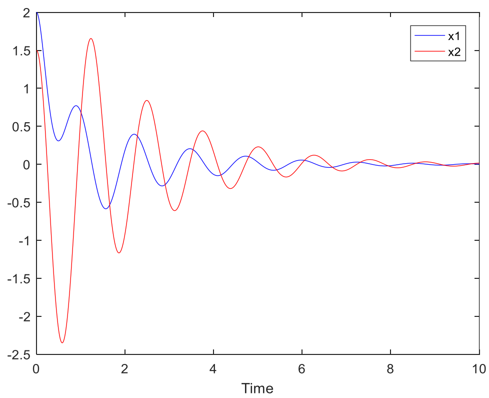

Figure 6: The output of x_1 and x_2 corresponding to $m_1 = m_2 = 2, k_1 = 24$; $k_2 = 24$; $k_3 = 24$; $b_1 = 2\sqrt{m_1 k_1}, b_2 = 0.001 * b_1$

Fourth case: Let us consider, $m_1 = m_2 = 2.6, k_1 = 30$; $k_2 = 30$; $k_3 = 24$: $b_1 = 2\sqrt{m_1 k_1}$, we also manipulate by changing $b_2 = 0.001 * b_1$. The numerical is as seen in the figure 7.

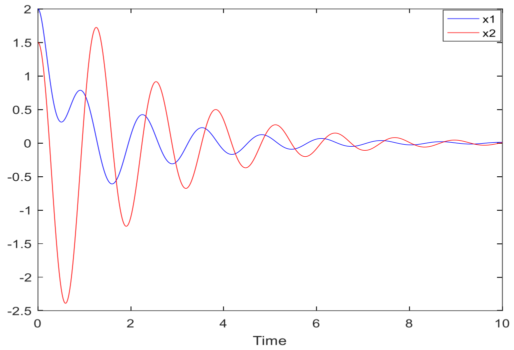

Figure 7: The Output of x_1 and x_2 corresponding to $m_1 = m_2 = 2.6, k_1 = 30;\ k_2 = 30;\ k_3 = 24$: $b_1 = 2\sqrt{m_1 k_1}$, $b_2 = 0.001 * b_1$

Although, we give both parameters b_2 and $b1$ different values by increasing decreasing we obtain just under damped in the third and fourth cases.

Fifth case: Let us suppose, $m_1 = m_2 = 1.5, k_1 = 1800;\ k_2 = 1800;\ k_3 = 1800$: $b_1 = 2\sqrt{m_1 k_1}$, we also manipulate by changing $b_2 = 0.001 * b_1$. Numerical is as seen in the figure 8.

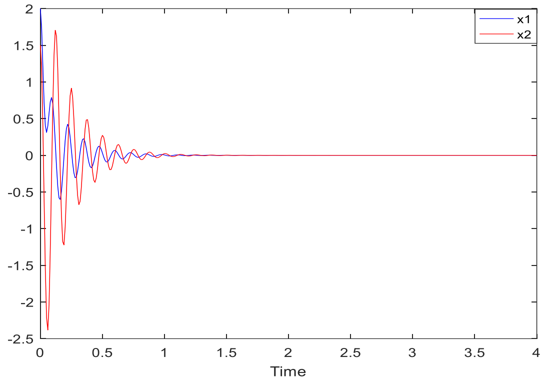

Figure 8: The output of x_1 and x_2 corresponding to $m_1 = m_2 = 1.5, k_1 = 1800;\ k_2 = 1800;\ k_3 = 1800$ and $b_1 = 2\sqrt{m_1 k_1}$, $b_2 = 0.001 * b_1$.

Although, we give both parameters b_2 and b_1 different values by increasing decreasing we obtain just under damped in this case.

Final case: Let us consider $m_1 = m_2 = 4, k_1 = 60;\ k_2 = 60;\ k_3 = 60$: $b_1 = 2\sqrt{m_1 k_1}$, we also manipulate by changing $b_2 = 0.001 * b_1$. Numerical is as seen in the figure 9.

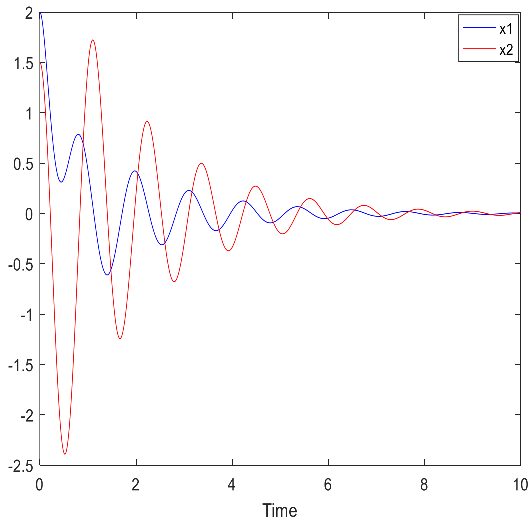

Figure 9: The output of x_1 and x_2 corresponding to $m_1 = m_2 = 4, k_1 = 60;\ k_2 = 60;\ k_3 = 60$: $b_1 = 2\sqrt{m_1 k_1}$, $b_2 = 0.001 * b_1$

We have to draw the conclusion about Critical diminished oscillation and supercritical oscillation and which are affected by the parameters b_1 and b_2.
The two parameters b_1 and b_2 are Damping constant that describes the strength of damping forces are called the damping canstant uses to determine the following

- Under damped oscillations: Damping constant <1
- Critically damped oscillations: Damping constant = 1
- Over damped oscillations: Damping constant >1

Where the damping constant $b_1 = 2\sqrt{m_1 k_1}$.
Remarks:
1- The larger value of b, the more quickly the amplitude decreases.
2- The frequency $\omega' = \sqrt{\frac{k}{m} - \frac{b^2}{4m^2}}$, is no longere qual the $\omega' = \sqrt{\frac{k}{m}}$
3- ω' becomes zero, b becomes so large.
4- when: $\frac{k}{m} - \frac{b^2}{4m^2} = 0 \Rightarrow b = 2\sqrt{mk}$ is condition for critical damp.
5- if b is greaat than $2\sqrt{mk}$ is condition of overdamp.
6- if b is less than $2\sqrt{mk}$ is condition of underdamp.
From the previous results we obtain the under damp, we could not get both citical and over damp, as a result of that we shhould go in new scenario as: If $m_1 = m_2 = 7, k_1 = 60;\ k_2 = 60;\ k_3 = 60$; also $b_1 = 2\sqrt{m_1 k_1},\ b_2 = 0.9 * b_1$. Numerical is as seen in the figure 10

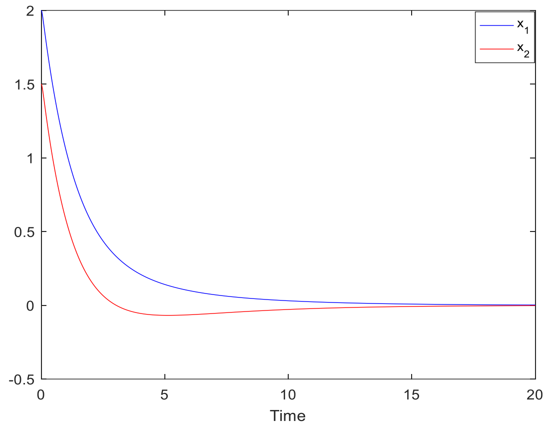

Figure 10: The output of x_1 and x_2 corresponding to $m_1 = m_2 = 7, k_1 = 60;\ k_2 = 60;\ k_3 = 60$: $b_1 = 2\sqrt{m_1 k_1},\ b_2 = 0.9 * b_1$

4. OTIMIZATION RESULTS

It is important to mention here that each harmonic oscillator of couple produces angular patterns as outputs for each joint. For estimating gait generation, it is very important to compute the optimal parameter sets for each harmonic oscillator. Of course, it is necessary to know how the angular position of both joints will change in time for generating rhythmic patterns. The parameter sets in the case $\{m_1, b_1, k_1, m_2, b_2, k_2, k_3\}$. The (GA) will be used to find the values of the parameters that optimize the objective function given below. The different walking patterns rely on this cost function:

$$J = -a_1 \sum_{k=1}^{n} x_b(k) + a_2 \left(\sum_{k=1}^{n} \left(\theta_1^2(k) + \theta_2^2(k) \right) \right) / N, \quad (8)$$

We assume that $a_1, a_2 \in [0,1]$ and $a_1 + a_2 = 1$, n is the number of elements of the position vector that is establishing patterns in order to maximize the displacement or velocity, and N is the length of time. When $a_2 = 0$, the energy consumption is ignored; hence, the displacement is maximized

The two constraints revealed here are $0 \le \theta_1,\ \theta_2 \le \pi$ due to physical reasons. In the present study, a hybrid function is used during the optimization process which runs after the GA terminates for improving the value of the fitness function. A hybrid function is a combination of two or more optimization algorithms that work together to improve the overall performance of the optimization process. In this case, the hybrid function uses a genetic algorithm (GA) and another optimization algorithm, which is run after the GA has completed its search. The GA is a search algorithm that is inspired by the process of natural selection. It uses a population of candidate solutions that undergo selection, crossover, and mutation to produce new candidate solutions that are hopefully better than the previous ones. The GA is a powerful optimization technique that can efficiently search for the global optimum in complex and multi-dimensional search spaces. After the GA has completed its search, the hybrid function runs another optimization algorithm to further refine the solution. The specific optimization algorithm used in the hybrid function will depend on the problem being solved and the characteristics of the search space. Some common optimization algorithms that can be used in a hybrid function include local search algorithms, simulated annealing, and particle swarm optimization. The idea behind using a hybrid function is that the GA can efficiently explore the search space and find a good solution, but it may not be able to converge to the global optimum. By using another optimization algorithm in conjunction with the GA, the hybrid function can refine the solution and potentially converge to the global optimum. This can result in faster and more accurate optimization results. Locomotion can be achieved with the couple harmonic oscillators, as shown in Figs 11, 12, 13 and 14.

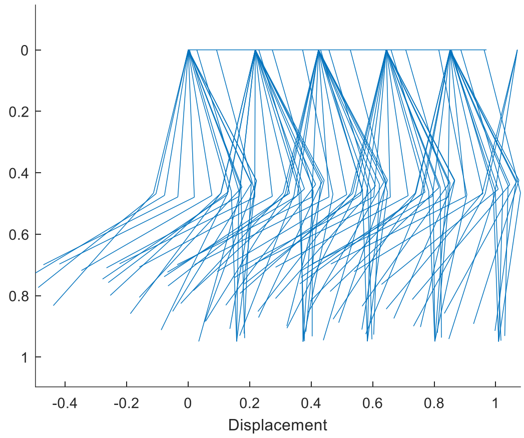

Figure 11: Animation of one leg for Couple harmonic oscillators: This animation corresponds to the values$m_1 = m_2 = 4, k_1 = 60;\ k_2 = 60;\ k_3 = 60$: $b_1 = 30.9839,\ b_2 = 0.31$.

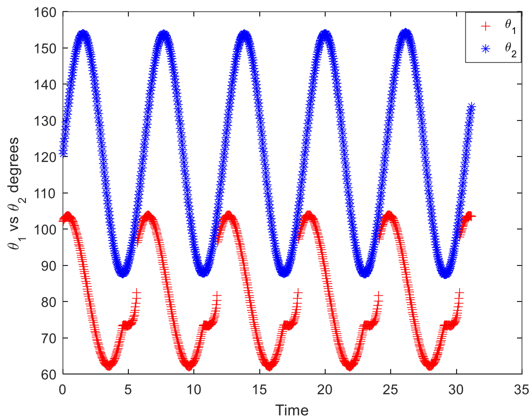

Figure 12: Angles against time: This solution corresponds to the same values in Figure 11.

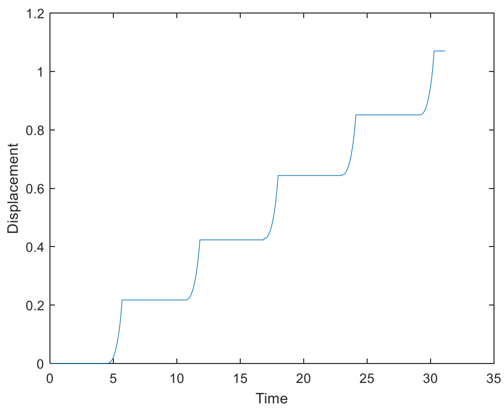

Figure 13: Displacement against time: This solution corresponds to the same values in Figure 11.

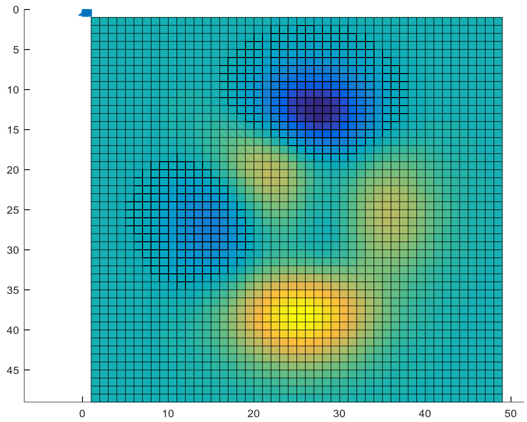

Figure 13: Peak surface against time: This solution corresponds to the same values in Figure 11.

5. CONCLUSIONS

This paper focuses on optimizing couple harmonic oscillators, the results show far greater compare with other studies. The couple harmonic oscillators produce the best performance level under damping case, which we obtain limit cycle. The results of the study warrant broader future applications of to continue research in different neural oscillators. These results are important, then not only because of what they may contribute to the ongoing discussion of locomotion but also for to see how to can use these results in upper body.

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